This surface is uniquely determined except for its position in space. The construction of the surface, viz., the determination of the Cartesian coordinates $x_i$ $(i=1,2,3)$, then proceeds by solving the fifteen scalar equations obtained from Eqs. (A1) and (A2) under the conditions

$$n\cdot n=1,\quad n\cdot r_{,\delta}=0(\delta=1,2),\quad r_{,\alpha}\cdot r_{,\beta}=g_{\alpha\beta}$$

$$n\cdot r_{,\alpha\beta}=b_{\alpha\beta},\qquad \alpha=1,2;\qquad \beta=1,2$$

However, it must be noted that in comparison to the aims of surface theory the aim of grid generation is to generate lines in a *given* surface. Despite the difference in aims, the basic equations of Gauss and Weingarten must always be satisfied.

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# Reduction of Component Mode Synthesis Formulated Matrices for Correlation Studies

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## Introduction

MANY of the published techniques for improving an analytical model or identifying areas of discrepancy in the model using measured test information requires reduction of the mass and/or stiffness matrices of the structure to the same degrees of freedom as measured during testing.[1-3] For structural systems that have a large number of degrees of freedom or have components designed by separate groups or organizations the method of component mode synthesis has proven to be an accurate, efficient, and economical method of analysis. The technique of component mode synthesis introduces generalized variables or Ritz coefficients that are not measurable or derivable quantities from testing into the final solution set. During a modal test of the actual structure, only physical variables are measured; therefore, it would be highly desirable to have a method that would operate directly on the final component mode synthesis system matrices by the elimination of generalized variables and any additional physical degrees of freedom not measured during testing while maintaining the same validity of the original system matrices. In this Note a technique for accomplishing this objective is described and subsequently applied to a numerical example.

## Theory

The final system equations from a component mode synthesis formulation using constraint modes may be partitioned into measured physical variables $U_a$, nonmeasured physical variables $U_o$, and generalized variables $U_q$:

$$\begin{bmatrix} M_{aa} & M_{ao} & M_{aq} \\ M_{oa} & M_{oo} & M_{oq} \\ M_{qa} & M_{qo} & M_{qq} \end{bmatrix} \begin{Bmatrix} \ddot{U}_a \\ \ddot{U}_o \\ \ddot{U}_q \end{Bmatrix} + \begin{bmatrix} K_{aa} & K_{ao} & K_{aq} \\ K_{oa} & K_{oo} & K_{oq} \\ K_{qa} & K_{qo} & K_{qq} \end{bmatrix} \begin{Bmatrix} U_a \\ U_o \\ U_q \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

The nonmeasured physical and generalized variables may be combined into a single vector of deleted variables $U_d$:

$$U_d = \begin{Bmatrix} U_o \\ U_q \end{Bmatrix} \tag{2}$$

Therefore, Eq. (1) may be rewritten in the following form:

$$\begin{bmatrix} M_{aa} & M_{ad} \\ M_{da} & M_{dd} \end{bmatrix} \begin{Bmatrix} \ddot{U}_a \\ \ddot{U}_d \end{Bmatrix} + \begin{bmatrix} K_{aa} & K_{ad} \\ K_{da} & K_{dd} \end{bmatrix} \begin{Bmatrix} U_a \\ U_d \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3}$$

where the entries in the mass and stiffness matrices are corresponding partitions of Eq. (1).

It is desired to express the $U_d$ solution variables, which are not measured during testing, in terms of the $U_a$ solution vari-

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ables, which are measured. At this point let us consider a modal transformation of the component mode synthesis solution variables given by

$$\begin{Bmatrix} U_a \\ U_d \end{Bmatrix} = \begin{Bmatrix} \Psi_a \\ \Psi_d \end{Bmatrix} \eta \tag{4a}$$

where $\Psi_a$ and $\Psi_d$ are partitions of the modal matrix truncated to the number of modes desired by the analyst, and $\eta$ is a vector of modal coefficients. From the top partition of Eq. (4a),

$$U_a = \Psi_a \eta \tag{4b}$$

Since $\Psi_a$ is normally rectangular, the direct inverse of $\Psi_a$ to solve for $\eta$ is not possible. Although a direct inverse cannot be obtained, $\eta$ can be obtained by the technique of pseudoinverse provided the columns of $\Psi_a$ are linearly independent. Assuming that independence of the columns of $\Psi_a$ does exist, the process may proceed by premultiplication of Eq. (4b) by the transpose of $\Psi_a$, giving

$$\Psi_a^T U_a = [\Psi_a^T \Psi_a] \eta \tag{5}$$

The bracketed matrix product in Eq. (5) is square of rank equivalent to the number of modes retained and is invertible. Therefore, solving for $\eta$,

$$\eta = [\Psi_a^T \Psi_a]^{-1} \Psi_a^T U_a \tag{6}$$

Now substituting Eq. (6) in the bottom partition of Eq. (4) gives

$$U_d = \{ \Psi_d [\Psi_a^T \Psi_a]^{-1} \Psi_a^T \} U_a = G_{da} U_a \tag{7}$$

Equation (7) provides the relationship between the measured and nonmeasured solution variables desired. This relationship has been previously used by Kammer[4] as an alternative to Guyan reduction for a full nonsubstructured model. The restriction on the validity of Eq. (7) is that the columns of $\Psi_a$ must be linearly independent when formed from the $U_a$ variables chosen for measurement. A necessary condition is that the number of measurement variables must be equal to or greater than the number of modes of interest. Normally, during modal testing the number of measurement points is much greater than the number of modes of interest; therefore, the likelihood of linear independence of the columns of $\Psi_a$ is enhanced.

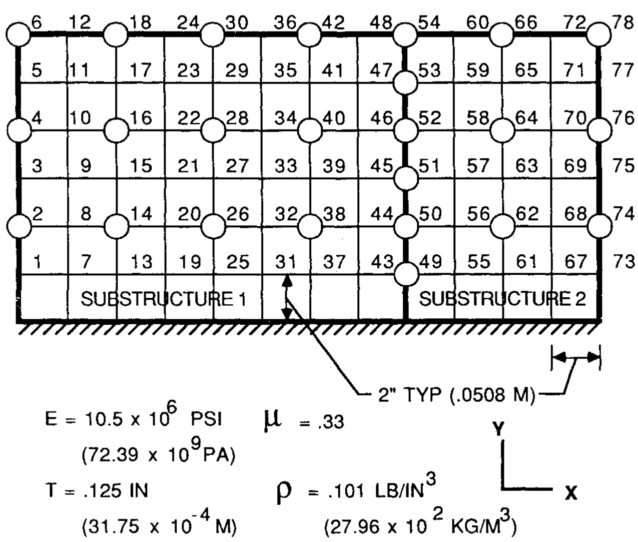


**Fig. 1 Rectangular cantilever plate.**

The total solution variables can now be expressed in terms of the measured variables $U_a$ by the following transformation:

$$\begin{Bmatrix} U_a \\ U_d \end{Bmatrix} = \begin{Bmatrix} I \\ G_{da} \end{Bmatrix} U_a = SU_a \tag{8}$$

In order to obtain the final reduced mass and stiffness matrices of solution variables only in terms of measured degrees of freedom, Eq. (8) is substituted in Eq. (3) and the subsequent equation premultiplied by the transpose of the reduction transformation matrix S to give

$$M_r \ddot{U}_a + K_r U_a = 0 \tag{9}$$

where

$$M_r = S^T M_s S, \qquad K_r = S^T K_s S \tag{10}$$

and $M_s$ and $K_s$ are the original unreduced system mass and stiffness matrix, respectively, in Eq. (3). The solution of Eq. (9) will provide the same frequencies and modes of displacement for those measured degrees of freedom as the original Eq. (1) or (3).

## Numerical Example

In order to demonstrate the application and validity of the reduction technique described in the previous section, a cantilever plate shown in Fig. 1 was analyzed. The plate is divided into two substructures along nodes 49–54. There are three degrees of freedom at each of the 78 nodes: rotations in the $x$ and $y$ directions and translation in the $z$ direction. This is the same example problem used by Craig and Bampton[5] in their oftened-referenced paper on component mode synthesis. The circled nodes in Fig. 1 were brought down to the final system representation. The total number of degrees of freedom in the system representation consisted of 102 variables: 72 physical variables from the 24 nodes carried down and 15 generalized variables per substructure representing the number of modes used to describe the dynamics of substructures 1 and 2.

At this point assume a modal test has been conducted in which only translation degrees of freedom are available to define the mode shapes. These are available for the circled nodes in Fig. 1 with the exception of 49, 51, and 53; this will result in 21 available measured solution variables. For the plate model shown in Fig. 1, the reduction technique previously described was applied using the first 15 analytical modes found. A comparison of the natural frequencies and mode shapes at measured degrees of freedom for the original 102-degrees-of-freedom component mode synthesis model and 21-degrees-of-freedom reduced model gave identical numerical results.

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